**UNIT III Special theory of relativity**

**Special theory of relativity:** Inertial and non-inertial frames, Galilean transformation, Michelson-Morley experiment, Einstein postulates of special theory of relativity, Lorentz transformation equation, length contraction, time dilation, variation of mass with velocity, Mass=energy relation.

Q. 5 Derive expression of length-contraction phenomena using Lorentz transformation equations.



In above fig. a rod is shown in a moving frame S’ (moving with constant velocity which is comparable to c). An observer in S’ observes its length as *l0* (called proper length). The co-ordinates of this rod are *x1’* and *x2’*. In stationary frame S, the observed co-ordinates are *x1* and *x2*. Where (*x2 –x1* = *l*).

Since the speed of moving frame is comparable to speed of light (c) therefore

Using Lorentz transformation equations, we can write

and

Therefore,

**Since v ˂ c then the quantity is always less then 1.** Thus measured length *l* of rod along the direction of motion is observed less than the *l*0. It shows that length observed by an observer in stationary fame seems contracted. Therefore, in terms of measurement of length *l* made in stationary frame, this phenomenon is called ***length-contraction.***

Numerical problem:

1. What will the length of a meter rod appear t be for a person travelling parallel to the length of the rod at a speed of 0.8c relative to the rod.
2. Show that the circle, x2+ y2= a2 in frame S appears to be an ellipse in frame S’ which is moving with velocity v relative to S.

[Hint: Here the changes occur in x direction only and y remain the same. Therefore, **.** Therefore**,** . Solve it.]

Q. 6 Derive the expression of time-dilation phenomena using Lorentz transformation equations.

Fig. 1 shows a spacecraft moving with comparable speed of light in x direction. At different instant of time an observer in the space craft observes the situation of photon traveling between two mirrors.

Here, the proper time (time as measured by an observer in spacecraft) *t0*=2*l0/c*

Fig. 2 shows the situation of the photon as seen by an observer in stationary frame (say on earth) or the path of light ray observed by an observer on earth).

From fig.

Since **proper time** *t0*=2*l0/c* or *t0*2 = 4*l02/c2*

Therefore,

Or

Since v ˂ c then the quantity is always less then 1. Thus time observed by an observer in stationaty fame (*t*) seems dilated (lengthened). Therefore, in terms of measurement of time t made in stationary frame, this phenomenon is called ***time dilation.*** Time dilation is ***a real effect.***

***Experimental verification:*** meson elementary particles are created at high altitude in earth atmosphere. But they are found on earth surface. This particle decays in electron or positron with a short life time of about 2.0x 10-6 sec. Therefore, distance travelled with this speed of light is d = v.t = 2.99x108 m/sec x 2.0 x 10-6 sec.

This life time (2.0x 10 -6 sec) of meson is in its own frame of reference. In the observer’s frame of reference on the earth surface, the life time of the meson is lengthened (dilated) due to relativity effects to the value t given as.

In this dilated life time, a µ-meson can travel a distance d0 = 2.99x108 m/sec x 3.17 x 10-5 sec = 9500 meter or 10 km.

This explains the presence of µ-meson on the earth surface despite of their short life time.

Numerical problem:

1. A man leaves the earth in a rocket ship that makes a round trip to the nearest star which is 4 light years away at a speed of 0.8c. How much younger will he be on his return that his twin brother who preferred to stay on earth.
2. [Hint: find t and t0 and then their difference.]